

# Mathematical programming for the optimal allocation of healthcare resources



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## BACKGROUND

The standard decision rules of cost-effectiveness analysis either require the decision maker

- to set a threshold willingness to pay for additional health care or
- to set an overall fixed budget.

In practice, neither are generally taken, but instead an arbitrary decision rule is followed which:

- may not be consistent with the overall budget
- may lead to an allocation of resources which is less than optimal
- is unable to identify the programme which should be displaced at the margin

## AIMS

We aim to show, using a policy-relevant example, how mathematical programming (MP) can be used as a generalisation of the standard decision rules.

This allows us:

- to examine alternative budgetary rules about when expenditure can be incurred,
- to show that indivisibility in a patient population and other equity concerns can be represented as constraints in the programme

## METHODS

The objective : to determine the optimal values of the available healthcare treatments ( $x_{jk}$ ) so as to maximise the gross benefit B

subject to an overall budgetary constraint  $\delta$ ,

and constraints that ensure all members of each independent healthcare programme k and population group  $i=1..I_k$  receive one and only one treatment  $j=1..J_k$

$$\max_{x_{jk}} \Psi = \sum_{t=1}^T C(t) \leq \delta$$

$$0 \leq x_{jk} \leq 1 \quad i=1..I_k, j=1..J_k, k=1..K$$

$$\sum_{j=1}^{J_k} x_{jk} = 1 \quad i=1..I_k, k=1..K$$

Total Health Benefits (QALYs) over T years, relative to current care

$C(t)$  are the costs at time t (relative to current care), that is:

$$C(t) = \sum_{i=1}^{I_k} \sum_{j=1}^{J_k} x_{jk} c_{ijk}(t) \quad t=1..T$$

Decision variables are the proportion of population i and healthcare programme k to receive treatment j

All members of each programme and population group receive one and only one treatment

## POLICY EXAMPLE

We demonstrate the method using data taken from the 6<sup>th</sup> and 7<sup>th</sup> wave of NICE appraisals. We assume all treatments must be provided within a fixed budget. We show results for a range of possible budgets.

## RESULTS

Figure 1: The shadow price (QALYs per additional £million) of the overall budget constraint at different values of the overall budget, and corresponding threshold cost per QALY

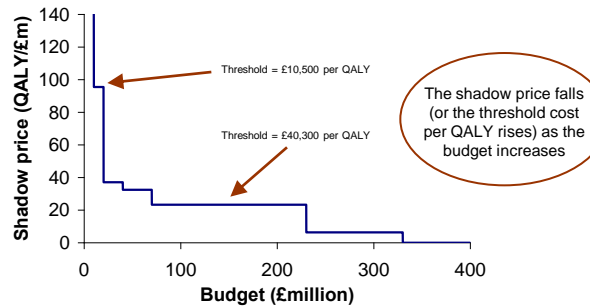


Table 1: QALY loss with alternative budget rules

Budget rule	Health gain (QALY)	Opportunity Loss (QALY)	Budget spent £m
No constraint	7317	0	£180
Equal phasing	3586	3731	£103
All in 1st 5 years	4879	2438	£75

Maximum health gain is obtained if there are no constraints about when the budget can be spent (for a budget of £180m)

If the total budget (£) is divided into equal sized maximum annual budgets over T=15 years, there is an opportunity loss compared with no restrictions, and not all the budget can be spent

If the budget must be spent in the first five years, and none thereafter, treatments that have costs beyond 5 years are only permitted if their costs are offset by others that are cost saving in those periods

Table 2: QALY loss with indivisibilities (horizontal equity)

	Health gain (QALY)	Opportunity Loss
No equity constraint	3586	0
Indivisibility in popn. 1	3066	520
Indivisibility in popn. 2	3547	19
Indivisibility in all patient populations	3066	520

Health gain for a budget of £180m and an 'equal phasing' budget constraint

We impose an equity rule that all members of a particular population group and healthcare programme are treated the same way

The effect of the equity concern is different in population 2

We impose the equity rule that all patients in each population group and healthcare programme are treated the same way. In this case, the additional constraints (other than indivisibility in population 1) are not binding

## CONCLUSIONS

We show that MP can be used to allocate resources to treatments within and between patient populations, using a policy-relevant example. The outcome is equivalent to the rules of Johannesson and Weinstein (1993), using a cost-effectiveness threshold, if one is willing to accept the resulting level of expenditure as the budget. MP is able to generalise these rules, to accommodate constraints such alternative budgetary rules about the timing of expenditure, or incorporate indivisibilities (and other equity concerns). We show the efficiency loss from these additional constraints and show that the effect of equity concerns will vary from patient population to population.